PG – 149

Third Semester M.Sc. Degree Examination, December 2015
(CBCS)
MATHEMATICS
M 302 T : Mathematical Methods

Time : 3 Hours   Max. Marks : 70

Instructions : *1) Answer **any five** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Obtain an equivalent integral equation for the differential equation :

$\phi''(x) - 3\phi'(x) + 2\phi(x) = 4\sin x$ with conditions $\phi(0) = 1,\ \phi'(0) = -2$.

b) Let K(x, y) be a symmetric and $\psi_m(x)$ and $\psi_n(x)$ be the eigen functions of K(x, y) for distinct eignvalues λ_m and λ_n, then prove that

$\int_a^b \psi_m(x)\psi_n(x)\,dx = 0$. Also prove that these eigen values are real. **(7+7)**

2. a) State and prove the Hilbert-Schmidt theorem.

b) Find the asymptotic expansion of the error functions as $x \to 0$ and as $x \to \infty$. **(7+7)**

3. a) Obtain the leading order form of $\int_{-\pi/2}^{\pi/2} (t+2)e^{-x\cos t}dt$ as $x \to \infty$.

b) Find the asymptotic expansion of integral $I(x) = \int_0^{\pi/2} e^{-x\sin t}\,dt$ as $x \to \infty$ using the Watson lemma. **(7+7)**

4. a) Derive the Runge-Kutta second and fourth order formula for the solution of $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$.

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b) Solve the boundary value problem : $\frac{d^2y}{dx^2} - xy = 0$ with $y(0) + y'(0) = 1$ and $y(1) = 1$ with $\Delta x = \frac{1}{3}$ using the finite difference scheme. **(7+7)**

5. a) Derive the predictor-corrector methods for the solution of an initial value problem $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$.

b) Derive the Schmidt method for $U_t = U_{xx}$ and hence obtain the first-time level solution with conditions :

$$U(x,0) = 1,\ 0 \le x \le 1$$

$$\left.\begin{aligned} \frac{\partial U}{\partial x}(0, t) &= U(0, t), \\ \frac{\partial U}{\partial x}(1, t) &= -U(1, t) \end{aligned}\right\} t \ge 0$$

With $\Delta x = \frac{1}{4},\ \lambda = \frac{1}{4}$. **(7+7)**

6. a) Solve the hyperbolic equation $U_{tt} = U_{xx},\ 0 \le x \le 1,\ t \ge 0$ with conditions :

$$U(x,0) = x(1-x),$$

$$\frac{\partial U}{\partial t}(x,0) = 0$$

$$U(0, t) = U(1, t) = 0,\ t \ge 0$$

Take $\Delta x = 0.25$, $\Delta t = 0.1$. Use explicit scheme and obtain the solution at second time level.

b) Solve the elliptic equation :

$U_{xx} + U_{yy} = \sin \pi x \sin \pi y,\ 0 \le x, y \le 1$ with conditions $U = 0$ on the boundary. Take $\Delta x = \Delta y = \frac{1}{3}$ and use five-point formula. **(7+7)**

7. Derive the alternating direction implicit scheme for the two-dimensional parabolic equation. Discuss its stability criteria. **14**

8. Solve the parabolic equation :

$U_t = U_{xx} + U_{yy}$, $0 \le x, y \le 1$ with conditions

$U(x, y, 0) = \sin \pi x . \sin \pi y, \ 0 \le x, \ y \le 1$

$\frac{\partial U}{\partial x}(0, y, t) = \frac{\partial U}{\partial x}(1, y, t) = 0$

$\frac{\partial U}{\partial y}(x, 0, t) = \frac{\partial U}{\partial y}(x, 1, t) = 0$

Take $\Delta x = \Delta y = \frac{1}{2}$, $\lambda = \frac{1}{8}$. Obtain the solution at first time level. **14**